Abstract—In this article the dynamics of a multimachine power system model is studied using bifurcation theory. The classical 9 bus WSCC power system model is considered. It is represented using a full differential ODE set, including the corresponding dynamics of the control loops and the transmission lines. The analysis is carried out using the load demands as bifurcation parameters, in order to reveal possible dynamical scenarios for different loading conditions. For variations of one of the loads, Hopf and saddle-node bifurcations were detected. Furthermore, the bifurcation analysis varying two loads simultaneously reveals the existence of a pair of double-Hopf and a zero-Hopf bifurcations, acting as organizing centers of the dynamics.

Index Terms—Multimachine power systems, bifurcation analysis, oscillations, double-Hopf and zero-Hopf bifurcations.

I. INTRODUCTION

Actual power systems are continuously approaching their stability limits due to an increasing power demand and other factors such as quality regulations, environmental and economic restrictions. In addition, the operating point of a power system network is continuously suffering perturbations and depending on their severance the system may become unstable. Instability might be caused by many different phenomena (or due to a combination of them), the most common are the changes in the load consumption that reaches the limits of the transmission network or the generation capacity, action of badly tuned controls such as automatic voltage regulators (AVR) and power system stabilizers (PSS), line and generator tripping, etc. Some countermeasures to improve the system robustness are the reinforcement of the transmission lines between the generation and load centers, improvement in control devices and real-time communications, decisions based on reliable information and system contingency plans such as emergency load shedding schemes [1].

Since a power system is essentially a non-linear system with a set of parameters, like controller gains, load parameters, etc., bifurcation theory seems to be a suitable approach to study its dynamics. There are several articles that introduce this topic [2]–[6], where a description of the fundamental implications of bifurcations like saddle-node and Hopf in the system stability are presented. One of the most popular benchmark to study bifurcations in power systems was introduced in [7], where the authors presented a simple 3-bus model including a variable load. This particular example has been widely studied by many researchers using different sets of parameter values (see [8] and references therein). The dynamics of this simple model has proved to be complex including oscillations, voltage collapse and chaos.

In this work, one and two parameter bifurcation analyses are carried out for a multimachine detailed model of the common 9 bus Western Systems Coordinating Council (WSCC). The system is formulated as an entirely differential ODE model composed by three generators with their corresponding control loops, and three aggregated loads formed by induction motors and static components. The dynamics of the transmission lines and stators are included to perform a bifurcation analysis using standard continuation packages like MATCONT. The underlying goal is to analyze the system behavior for different loading scenarios and to show that useful information, related to instabilities and oscillations can be extracted using bifurcation theory. Additional information, not evident from a classical linear analysis, can be obtained performing a bifurcation study varying two parameters simultaneously. For example, it is possible to determine regions of qualitative different dynamics and find organizing centers where bifurcation branches emerge. There are few papers that perform a multiparameter bifurcation analysis in multimachine power systems, some examples are [9] and [10], but they are oriented to the use of bifurcation diagrams to find appropriate values of control parameters, like the AVR gains.

This work is organized as follows. In Section II the multimachine power system model is described. In Section III the dynamical behavior of the system varying one of the loads is studied using the bifurcation diagram. Then, in Section IV, a two parameter bifurcation study varying two loads simultaneously is performed. A description of some organizing centers of the dynamics is also presented in this section. Finally, concluding remarks are given in Section V.

II. MULTIMACHINE POWER SYSTEM MODEL

The analysis presented in this work is based on the 9-bus model of the WSCC power system represented schematically in Fig. 1 [11], [12]. It is composed by three generators, one hydro unit (#1) and two steam units (#2 and #3) with their respective voltage controls (Automatic Voltage Regulator - AVR) and a frequency regulator (Governor - GOV) placed only on generator #1. There are three composite loads with static and dynamic characteristics at buses 5, 6 and 8. The set of equations used in this article are described in the following subsections. All the variables are in per unit. The definition of the state variables and the parameter values are listed respectively in Tables II and III of Appendix A.
A. Synchronous Generators

The mathematical model of the generators is a 6-th order ODE set that considers the mechanical and electrical behavior of the rotor and the stator dynamics. The later ensures compatibility between the generator model and the network dynamics, presented in subsection II-D. The model is valid for both hydraulic and steam-turbine generators, and the equations for the $i$-th ($i = 1, 2, 3$) generator are,

$$\dot{\delta}_i = \Omega_B (\omega_i - \omega_s),$$

$$2H_i \dot{\omega}_i = P_{mi} - P_{ei} - K_{di} (\omega_i - \omega_s) - D_{bi} \omega_i,$$

$$T_{d0}' e_{q'i} = -e_{q'i} + (X_{di} - X_{d'i}) i_{di} + e_{f'di},$$

$$T_{q0}' e_{di} = -e_{di}' - (X_{qi} - X_{q'i}) i_{qi},$$

$$X_{qi} T_{q0}' \dot{i}_{qi} = -e_{di}' - i_{qi} (X_{qi} - X_{q'i}) - T_{q0}' \Omega_B (v_{qi} - i_{di} X_{di}' - e_{q'i} + i_{qi} R_{si}),$$

$$X_{di} T_{d0}' \dot{i}_{di} = e_{q'i} + e_{f'di} - i_{di} (X_{di} - X_{d'i}) - T_{d0}' \Omega_B (v_{di} + i_{qi} X_{q'i}' - e_{di}' + i_{di} R_{si}),$$

with $P_{ei} = e_{q'i} i_{qi} + e_{di}' i_{di} + (X_{di}' - X_{d'i}) i_{qi} i_{di}$.

B. Excitation Systems

Two types of normalized excitation systems and AVR will be considered for the generators. The first is the IEEE-DC1A DC Excitation System, for generator #1. This is a slow response DC excitation/AVR system. The second one is the IEEE-ST1A+PI Static Excitation System, for generators #2 and #3. This excitation/AVR model is based on static elements and it has a fast dynamic response [13]. The block diagrams of these control systems are shown in Fig. 1 and the corresponding equations are not included here since they are simple to extract from the diagrams.

C. Prime-Mover Turbine and Governor

The dynamical behavior of the hydraulic turbine attached to generator #1 is represented by a nonlinear model used for transient stability studies [14]. The linear dynamic of the servomotor driving the gate of the turbine is also considered. The governor unit used for primary and secondary frequency regulation is a PI controller, and a transient term is added to compensate the non-minimum phase effect introduced by the turbine. The equations of the turbine-governor can be obtained from the block diagram included in Fig. 1.

D. Transmission Network

In the traditional DAE model of a multimachine power system, the dynamics of the transmission network is considered so fast that it is neglected [15]. This model is difficult to use in bifurcation studies, since the most common continuation packages, such as AUTO [16] or MATCONT [17], do not support this type of representations. Although there are algorithms based on continuation methods developed for
tracking equilibria bifurcations in DAE systems [18], [19], they do not offer additional features such as continuation of periodic solutions and their bifurcations. Therefore to make the power system model compatible with the classical continuation packages, a fully differential ODE model of the multimachine power system must be derived. Toward this end the fast dynamics of the currents and voltages of the transmission network are considered [4], [20]. It is important to notice that all the transmission system variables are referred to a synchronous rotating DQ frame. The lines are represented by the lumped parameter π equivalent circuit shown in Fig. 2. The values of the network components are indicated in Fig. 1. Then, for each transmission line there are two differential equations describing the current

$$\Omega_B^{-1} I_{nm} \dot{i}_{Qnm} = v_Q - v_m - R_{nm} i_{Qnm} + \omega_s L_{nm} i_{Dnm}, \quad (7)$$

$$\Omega_B^{-1} I_{nm} \dot{i}_{Dnm} = v_D - v_m - R_{nm} i_{Dnm} - \omega_s L_{nm} i_{Qnm}, \quad (8)$$

In addition, there are two differential equations for the dynamics of the bus voltages (excluding generation buses)

$$\Omega_B^{-1} C_n \dot{v}_Q = i_{Qcn} + \omega_s C_n v_{Dn}, \quad (9)$$

$$\Omega_B^{-1} C_n \dot{v}_D = i_{Dcn} - \omega_s C_n v_{Qn}, \quad (10)$$

where $i_{Qcn}$ and $i_{Dcn}$ are the DQ components of the current through the capacitor $C_n$, obtained from the node equation

$$i_{Cn} = i_n - i_{nm} - i_L, \quad (11)$$

where $i_L$ is the current in the load (if any), $i_{nm}$ is the current between the nodes $n$ and $m$ and $i_n$ represents all other currents involved in node $n$. Notice that, since the currents of the generators are expressed in their own frame, they must be transformed to the synchronous DQ frame (as shown in Fig. 3) before being considered in $i_n$.

E. Composite Load Model

The load modeling is a critical issue in power system analysis, since the stability properties of a given network is strongly dependent on the load considered. This is an extremely complex subject that is still under investigation [21]. In this paper, following the recommendation of [22]–[24], composite loads consisting of static and dynamic elements are used. The static portion is a constant ZIP (impedance (Z), current (I) and power (P)) load. Then, the active and reactive power drawn by the j-th ($j = 5, 6, 8$) load are,

$$P_j = \alpha_j \left( P_j Z_j \frac{v_j^2}{V_0^2} + P_j I_j \frac{v_j}{V_0} + P_{Qj} \right), \quad (12)$$

$$Q_j = \beta_j \left( Q_j Z_j \frac{v_j^2}{V_0^2} + Q_j I_j \frac{v_j}{V_0} + Q_{Qj} \right), \quad (13)$$

where, $v_j = (v_{Qj}^2 + v_{Dj}^2)^{1/2}$, is the voltage magnitude of the corresponding load bus (referred to the synchronous DQ frame). The DQ components of the current drained by the ZIP load should be obtained from (12) and (13) to calculate the corresponding part of the load current $i_L$ in (11).

The dynamic portion of the load is given by induction motors with a two axis transient model including the stator dynamics. The differential equations are

$$2H_j \dot{\omega}_j = \alpha_j (2A_j^2 + B_j \omega_j + C) P_{mj} \quad (14)$$

$$- P_{ej} - D_{ej} \omega_j, \quad (15)$$

$$T_{dq_{0j}}' \dot{e}_{qj} = -e_{qj} + (X_{dqj} - X_{dqj}') i_{dj} - T_{dq_{0j}}' \Omega_j (\omega_j - \omega_s) e_{qj}, \quad (16)$$

$$T_{dq_{0j}}' \dot{e}_{dj} = -e_{dj} - (X_{dqj} - X_{dqj}') i_{qj} + T_{dq_{0j}}' \Omega_j (\omega_j - \omega_s) e_{qj}, \quad (17)$$

$$X_{dqj} T_{dq_{0j}}' \dot{i}_{qj} = T_{dq_{0j}}' \Omega_j (X_{dqj} i_{qj} + e_{qj}' - v_{qj}) \quad (18)$$

$$+ i_{dj} (X_{dqj} - X_{dqj} - R_{ej} T_{dq_{0j}}' \Omega_j) - e_{qj}', \quad (19)$$

$$X_{dqj} T_{dq_{0j}}' \dot{i}_{dj} = T_{dq_{0j}}' \Omega_j (X_{dqj} i_{qj} + e_{qj}' - v_{qj}) \quad (20)$$

$$+ i_{qj} (X_{dqj} - X_{dqj} - R_{ej} T_{dq_{0j}}' \Omega_j) - e_{qj}', \quad (21)$$

with $P_{ej} = e_{qj}' i_{qj} + e_{dj}' i_{dj}$. The parameter $\alpha_j$ is introduced to vary in equal proportions the power consumption of the static and the dynamic parts of the loads [see Eqs. (12) and (14)]. This parameter indicates the level of power demand, and $\alpha_j = 1$ corresponds to the nominal operation point. The parameter $\beta_j$ has a similar function, but for the reactive power demand. Notice that the induction motor do not have angle dynamics and thus the variables are in the synchronous DQ frame and are directly added in the total load current $i_L$.

III. ONE PARAMETER BIFURCATION ANALYSIS

The nominal operating point of the WSCC model, i.e. the power transmitted by the generators and the power drained by the loads is obtained from [11], and indicated in Fig. 1. This point is locally stable and it should be robust enough to
overcome typical perturbations, such as small variations in the demand of the loads.

In this work the generation units are re-dispatched in order to analyze a stressed operating condition. This is a common practice on a real system due to changes in the load demand, security reasons, economic considerations, physical restrictions or emergency conditions. It is important to mention that the re-dispatch can lead to oscillatory instability [25], [26]. In the following, the mechanical power of generator #2 is set near the maximum value allowed ($P_{m2} = 1.90$ p.u.), the generation unit #1 will diminish the power generated due to the governor action, and the set point of generator #3 is left unchanged. The re-dispatch of generators is indicated in Table I.

The dynamical behavior of the power system is analyzed in terms of variations of the active power of the load at bus 5, i.e. considering variations of parameter $\alpha_5$. The behavior of the magnitude of the voltage at the load bus 5 ($|V_5|$) is depicted in the bifurcation diagram of Fig. 4. Solid lines represent stable operation conditions, while the unstable points are shown with dashed lines. The filled dots represent the amplitude of stable oscillations (limit cycles) and unfilled dots are unstable periodic solutions.

Let us now describe the bifurcation diagram. Decreasing the load on bus 5 below the nominal value ($\alpha_5 = 1$) the equilibrium point becomes unstable due to a Hopf bifurcation, namely $H_1$ in Fig. 4a. Consequently, a pair of complex eigenvalues of the linearization crosses the imaginary axis (and the non-degeneracy conditions are satisfied [27]) for $\alpha_5 = 0.9050$. Hence, the equilibrium point becomes unstable and a stable limit cycle (sustained oscillation) emerges towards the left of $H_1$, i.e. for decreasing values of $\alpha_5$. Notice that even though the linearization around the equilibrium has a pair of unstable complex eigenvalues, there is a stable oscillation that may prevent the system to diverge.

When the load on bus 5 is increased from its nominal value, the bus voltage begins to decrease. Then, after a significant increment of $\alpha_5$, the equilibrium point becomes unstable at $\alpha_5 = 2.1282$ due the Hopf bifurcation $H_2$ shown in Fig. 4b (blow up of the rectangle in Fig. 4a). The equilibrium recovers stability after the Hopf bifurcation $H_3$ located at $\alpha_5 = 2.1289$. Afterwards it collides with the saddle point (denoted with dashed lines in Fig. 4a) and both disappear in a saddle-node bifurcation $LP$. Beyond this value of $\alpha_5$ there are no possible operating points.

The unstable limit cycle born at $H_2$ collides with the stable cycle created at $H_3$ in a cyclic fold bifurcation $CF$ for $\alpha_5 = 2.1280$. Hence, for values of $2.1280 < \alpha_5 < 2.1282$ there are two nested limit cycles and, then, a bistability competition between a stable equilibrium and a stable periodic solution arises depending on the initial conditions. This special structure of cycles (Fig. 4b) is confined to the neighborhood of $LP$, where the dynamics is very influenced by the saddle equilibrium, restricting the basin of attraction of the stable equilibrium or even the stable cycles. From a one parameter analysis it is not evident that for variations of other parameters this oscillatory behavior could not appear in regions of practical importance. Therefore, it is important to recognize the differences between bifurcations like $H_1$ or $H_3$ (supercritical) and those like $H_2$ (subcritical), since it might be useful in cases where a similar structure of nested cycles like the one shown in Fig. 4b appears. For example, if the operating point is on the left of $H_2$ and the load is increased until $H_2$ is crossed, the system will exhibit a “large” amplitude stable
oscillation. Then, if the load is decreased beyond $H_2$ the oscillation persists until $CF$ is reached (hysteresis). This is not the case in oscillations like $H_1$ or $H_3$. This information can not be deduced from a linear stability analysis.

### A. Eigenvalue Movement

Since the bifurcations of equilibria seen in the preceding section (Hopf and saddle-node) are local phenomena, it is interesting to study the eigenvalues associated to the equilibrium and their movement when the parameter $\alpha_5$ is varied in the range used in Fig. 4a. The movement of the relevant eigenvalues as $\alpha_5$ is increased is shown in Fig. 5 (only the positive imaginary parts are shown and the arrows indicate the movement direction). The analysis can be improved calculating the participation factors [12], [15], to determine which state variables are the main responsible for the occurrence of bifurcations. Then starting from $\alpha_5 = 0.8$ there is a pair of unstable complex eigenvalues at $0.016 \pm i8.563$, while the rest of the eigenvalues are in the left half plane. When $\alpha_5$ is increased the unstable pair moves toward the left and crosses the imaginary axis at $0 \pm i8.547$, developing the Hopf bifurcation $H_1$ (see Fig. 5). The participation factors near this singular point reveal a strong influence on the state variables $\delta_2$ and $\omega_2$, and thus the instability of the equilibria and the resulting oscillation due to the Hopf bifurcation $H_1$ can be associated to an angle instability.

Increasing the load on bus 5 further more, the previous angle oscillatory mode does not become unstable again, but a pair of eigenvalues related to the excitation system of generator #1 moves toward the imaginary axis. More precisely, the dominant state variables are $e_{q1}'$ and $r_{F1}$. The first crossing of these eigenvalues is at $0 \pm i0.894$ and determines the Hopf bifurcation $H_2$. The equilibrium remains unstable until the same pair crosses the imaginary axis again at $0 \pm i0.729$ determining the Hopf bifurcation $H_3$. Then the equilibrium becomes stable again. These phenomena are related to voltage stability problem. Notice that this mode describes the ellipsoid shown in Fig. 5.

Then, a real eigenvalue becomes unstable for a higher value of $\alpha_5$, leading to the saddle-node bifurcation $LP$ in Fig. 4a. The participation factors reveal that the state variable of the swing equation of the motor at bus 5 is dominant in this bifurcation.

### IV. Two Parameter Bifurcation Analysis

There are several advantages in performing a bifurcation analysis varying two parameters. For example, regions of qualitative different behavior can be found by tracing the codimension one bifurcation curves. In addition, it brings the possibility to study codimension two bifurcations that in a two parameter plane might be seen as organizing centers of the dynamics. Non-trivial phenomena may also be revealed studying the normal form of such codimension two singularities. In this section the dynamical aspects of the system will be studied when two loads are varied simultaneously.

As seen before, the Hopf bifurcation $H_1$ is caused by an angle instability associated to the generator #2 swing equation due to a variation in the load on bus 5. Then, it is interesting to study how this oscillation mode behaves when two loads are varied simultaneously. Similar arguments are valid for the other bifurcations shown in Fig. 4. To accomplish this task, parameter $\alpha_8$, i.e. the power drained by the load on bus 8 (see Fig. 1), is varied (decreased). At the same time $\alpha_5$ is varied satisfying the Hopf bifurcation condition and thus curve $H_1$ in Fig. 6 is obtained. Analogously, the bifurcation curves for $H_2$, $H_3$ and $LP$ are obtained by continuation of the corresponding bifurcation conditions. The curves have been computed using the continuation package MATCONT [17].

Some conclusions about the system behavior can be drawn from the Hopf bifurcation curves provided by the two parameter bifurcation scheme shown in Fig. 6a. One of the most important facts to be remarked is that the angle instability is the main problem, since the bifurcation curve $H_1$ exists for a wide range of values of $\alpha_5$ and $\alpha_8$, and the remaining bifurcation curves are confined at values of $\alpha_5 \approx 2.1$, regardless the value of $\alpha_8$. Notice that the relative distances of the phenomena shown in Fig. 4b (curves $H_2$, $H_3$ and $LP$ in Fig. 6) are practically not affected by the variation of $\alpha_8$. This means that the limit imposed by the saddle-node bifurcation $LP$ is virtually unaffected by the consumption of the load on bus 8 (the saddle-node is associated to a mechanical state of the load motor in bus 5).

The grey area enclosed by the curves $H_1$ and $H_2$ in Fig. 6 denotes a region where stable equilibria exist. Caution should be taken regarding the robustness of the equilibrium as the parameter is close to any of the bifurcation curves. Also notice that this is not the only area with stable equilibria since there is another one enclosed by the curves $H_3$ and $LP$. This area has no practical meaning because it is very narrow and therefore is not colored for the sake of simplicity.

Suppose that the system is in a feasible operating condition (grey area), and the load on bus 5 is constant ($\alpha_5$ fixed).
Then, if the load on bus 8 (parameter $\alpha_8$) is decreased and crosses the curve $H_1$, the system will experience an angle oscillation. Also, setting $\alpha_8$ constant and increasing $\alpha_5$, the instability will be caused by a voltage phenomenon such as the Hopf bifurcation $H_2$. In addition, keeping $\alpha_8$ constant and decreasing $\alpha_5$ it also leads to the angle instability due to $H_1$.

A. Organizing centers of the dynamics

Figure 6b offers an expanded view of the rectangle shown in Fig. 6a. Even though the parameter range is very small, this area contains a collection of codimension two singularities and concentrates the organizing centers of the dynamics.

1) Double Hopf Bifurcation: The intersection of the curves $H_1$ and $H_2$ defines the double Hopf (or Hopf-Hopf) bifurcation $HH_1$ in Fig. 6b and establishes the point in the parameter plane where both Hopf singularities occur simultaneously. In addition, the intersection of the curves $H_1$ and $H_3$ denotes another double Hopf point, namely $HH_2$ in Fig. 6b. Double Hopf bifurcations were studied in several engineering applications, such as electrical circuits [28]–[30], mechanical systems [31], [32], and neural systems [33]. The special resonant cases are studied in [34], [35].

The double Hopf singularity is a highly complex codimension two bifurcation with many possible scenarios. The normal form predicts eleven different unfoldings (six of them include non-trivial global phenomena), i.e. with different local dynamical phenomena [36]. These unfoldings are distinguished by the values of the normal form coefficients. A procedure for obtaining these coefficients (and those of the normal form for all the bifurcations shown in this paper), is explained in detail in [36] and it is implemented in the continuation package MATCONT.

A common factor in all the double Hopf scenarios is the presence of quasi-periodic oscillations (2D tori), involving two frequencies or modes. In the neighborhood of the Hopf-Hopf point these frequencies correspond to that of the limit cycles associated to the Hopf bifurcations, since the torus is born at local bifurcations (Neimark-Sacker) of the limit cycles. The case denoted as $HH_1$ has additional non-trivial phenomena such as heteroclinic orbits and a 3D torus with three main frequency components. The unfolding is depicted schematically in Fig. 7, where the Neimark-Sacker bifurcations are denoted as $TR_1$ and $TR_2$. The continuation software brings the possibility to continue the Neimark-Sacker curves varying two parameters, but in this case it demands a big computational burden due to the high dimension of the model (70 state equations). The quasi-periodic solutions are within the area enclosed by $TR_1$ and $TR_2$. In addition, the 3D torus can be found between the curve $C$ that represents a bifurcation of the 2D torus [37], and the curve $Y$ which is a heteroclinic connection.

Analyzing the normal form it can be determined that the region with tori for the $HH_1$ singularity is located to the left of $HH_1$ (locally), between $H_1$ and $H_2$ curves, enclosed by two (not shown) Neimark-Sacker bifurcation curves. In the $HH_2$ case, the unfolding corresponds to one of the simple cases that includes a single 2D torus, besides the limit cycles.
emanated from the Hopf bifurcations.

2) Zero Hopf Bifurcation: The singularity $ZH$ shown in Fig. 6b is a zero-Hopf (Gavrilov-Guckenheimer) bifurcation. At this point, the Hopf bifurcation curve $H_1$ becomes tangent to the saddle-node curve $LP$. Therefore, the linearization of the equilibrium has a pair of complex conjugated eigenvalues and a single eigenvalue with zero real part, denoting the simultaneous occurrence of the Hopf bifurcation $H_1$ and the saddle-node $LP$. This bifurcation can be interpreted as the generator of the saddle-node bifurcation shown in Fig. 4, since it corresponds to a point of the saddle-node curve emanating from the $ZH$ singularity. The normal form of this bifurcation is also described in [36], and in this case the corresponding unfolding has a quasi-periodic orbit in a neighborhood of the singularity. There is a high chance that this quasi-periodic orbit is in direct relation with the tori emanated from the Hopf-Hopf singularities. This fact might be confirmed by continuation of the Neimark-Sacker curve associated to $ZH$.

V. CONCLUSIONS

In this paper the dynamics of a detailed multimachine power system was studied using bifurcation theory and classical tools, such as the participation factors. The combination of these techniques gives an improved knowledge of the system behavior and the possible problems coming from the variation of system parameters. For example, using the bifurcation analysis it is possible to determine the specific values of the parameters where the system equilibrium becomes unstable due to a Hopf bifurcation, and also find out the stability of the associated oscillation. While using the participation factor it is possible to get information about the problem nature, i.e. the state variables responsible for such instability, in order to take the appropriated countermeasures.

The bifurcation analysis varying two parameters brings additional information such as the location of organizing centres of the dynamics like the double Hopf and zero-Hopf bifurcations. Then, studying the normal form of these codimension two points it is possible to explain some non-trivial phenomena like quasi-periodic motions. In addition, feasibility regions in the parameters space can be determined. Among the drawbacks found using continuation methods in power systems is the computational burden demanded to trace the bifurcation curves (specially the torus continuation), mostly due to the high dimension of the system. Even though, the continuation algorithms are very robust if their parameters are set correctly.

APPENDIX A

The definition of the variables and parameters of the system are listed in Table II and Table III, respectively. All values are in per-unit, using a 100 MVA base, otherwise the corresponding unit is indicated in parenthesis.

REFERENCES


